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Microarticle

On-axis intensity of hollow beams in nonlinear media with a high nonlocality

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ABSTRACT

In this paper, we focus on the on-axis intensity evolutions of hollow beams in nonlinear media with a high nonlocality. It is found that, the evolutions of on-axis intensity with different beam powers are always periodic during propagation, whereas, depending on different beam powers, the evolution curves of on-axis intensity in each period may exhibit themselves as three different types, namely, a concave, a platform, or a convex shape. The critical power, the extreme values of on-axis intensity and their corresponding positions are all given analytically.

The hollow beam (HB) with a zero central intensity at the initial plane is a type of important beams which have many potential and practical applications. A typical model of HBs has been proposed mathematically by Cai et al. [1], and they have been generated in experiments by various techniques [2–4]. HBs have been widely investigated in various optical media and systems [5–10]. On the other hand, highly nonlocal nonlinear media (HNNM) (i.e. these nonlinear media with a highly spatial nonlocality) have also attracted a lot of attention in the past decade [11]. The nonlocality sustains various stable and complex spatial solitons [11–15]. Recent years, laser beams propagating in HNNM have attracted intensive attention [16–19], and some distinctive properties are found [20–23]. For beam propagation, the on-axis intensity is an important characteristic in practical applications. In this paper, the on-axis intensity of HBs is investigated in HNNM in detail. It is found that, depending on different beam powers, the evolution curves of on-axis intensity could be a concave, a platform, or a convex shape.

A laser beam propagating along z direction in HNNM can be phenomenologically governed by the simplified nonlocal nonlinear Schrödinger equation, i.e. the Snyder–Mitchell model (SMM) [11,12], which can be rewritten in cylindrical coordinates as $2ik\partial_z\Phi + \partial_r\Phi/r + \partial_r^2\Phi + \partial_\theta^2\Phi/r^2 - k^2\gamma^2P_0r^2\Phi = 0$, where $\gamma^2 > 0$ is a material constant associated with the nonlocal response function of the media, and corresponds to the case of self-focusing. k is the wave number in the media without nonlinearity. $P_0 = \int_0^{2\pi}\int_0^{\infty}|\Phi|^2 r\,dr\,d\theta$ is the beam power. Because the SMM is a linear model, the propagation expression of a laser beam in HNNM can be obtained utilizing an integrate formula [22,23]. In cylindrical coordinates, the integrate formula can be expressed as $\Phi(r,\theta,z) = -\frac{ik\sqrt{\gamma^2P_0}}{2\pi\sin\left(\sqrt{\gamma^2P_0}z\right)}\exp\left[\frac{ik\sqrt{\gamma^2P_0}r^2}{2\tan\left(\sqrt{\gamma^2P_0}z\right)}\right]\times \int_0^{2\pi}\int_0^{\infty}\Phi(r_0,\theta_0)\exp\left[\frac{ik\sqrt{\gamma^2P_0}r^2}{2\tan\left(\sqrt{\gamma^2P_0}z\right)} - \frac{2ik\sqrt{\gamma^2P_0}rr_0\cos(\theta-\theta_0)}{\sin\left(\sqrt{\gamma^2P_0}z\right)}\right]r_0\,dr_0\,d\theta_0$.

The optical field of a HB on the initial plane is expressed as [1] $\Phi(r_0,\theta_0) = C_0\left[r_0^2/(2w_0^2)\right]^n\exp\left[-r_0^2/(2w_0^2)\right]$, where $C_0 = 2^n\sqrt{P_0/[2\pi\Gamma(2n)w_0^2]}$ is a normalized constant which ensures that the beam power is equal to P_0, and $\Gamma(\cdot)$ is the Euler gamma function. $n = 0, 1, 2, \ldots$ is the beam order. w_0 is the beam waist width of a fundamental Gaussian beam.

Substituting the expression of initial optical field of HBs into the above integrate formula, after a somewhat tedious process, one can obtain the propagation expression of HBs in HNNM [8,22,23]. And then, letting $r = 0$, the on-axis intensity expression can be obtained as $I(z) = (C_0n!)^2(P_{cg}\sin^2\alpha)^nP_0/(P_{cg}\sin^2\alpha + P_0\cos^2\alpha)^{n+1}$, where $\alpha = \sqrt{P_0/P_{cg}}z/z_R$, and $P_{cg} = 1/(\gamma^2z_R^2)$ is the soliton power of a fundamental Gaussian in HNNM [11,12], with $z_R = kw_0^2/2$ being the Rayleigh distance.

We now discuss the on-axis intensity evolution of HBs in HNNM. The evolution is always periodic, and it can be divided into three different types depending on the beam power. The critical

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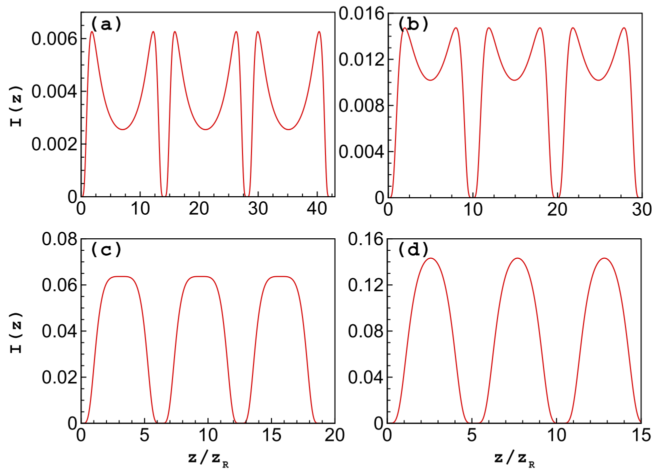

Fig. 1. On-axis intensity evolutions of HBs with different beam powers in HNNM. The beam power is $P_0 = 0.2P_{cp}^{(3)}$ for (a); $P_0 = 0.4P_{cp}^{(3)}$ for (b); $P_0 = P_{cp}^{(3)}$ for (c); $P_0 = 1.5P_{cp}^{(3)}$ for (d). The beam order is $n = 3$ for all plots.

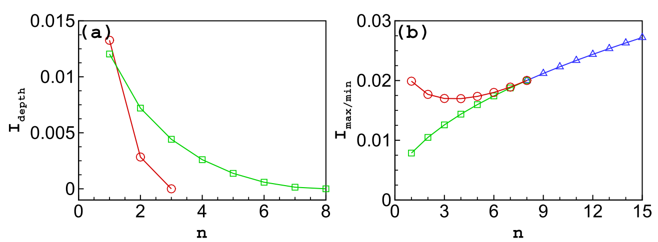

Fig. 2. Effect of the beam order on the on-axis intensity of HBs in HNNM. (a) The depth of the concave versus the beam order for a fixed beam power. The beam power is $P_0 = P_{cg}/4$ for the circles and $P_0 = P_{cg}/9$ for the squares. (b) The circles and the squares denote, respectively, the maximal and minimal on-axis intensity for the concave case. The triangles denote the maximal intensity for the convex case. The beam power is fixed at $P_0 = P_{cg}/9$.

beam power is $P_{cp}^{(n)} = P_{cg}/(n+1)$. It is evident that the critical beam power decreases with the beam order increasing.

(a) The concave case. When $P_0 < P_{cp}^{(n)}$, the evolution curve of on-axis intensity manifests itself in a concave in each period (see Figs. 1(a) and 1(b)). In each concave/period, there exist three extreme values, i.e. two maximal values and one minimal value. The three extreme values are located at, respectively, $z_{max} = \left[q\pi \pm \frac{1}{2}\arccos\left(1 + \frac{2nP_0}{P_0 - P_{cg}}\right)\right]\left(\frac{P_{cg}}{P_0}\right)^{\frac{1}{2}} z_R$ and $z_{min} = (2q+1)\frac{\pi}{2}\left(\frac{P_{cg}}{P_0}\right)^{\frac{1}{2}} z_R$, where $q = 1, 2, 3, \cdots$. The values of maximum and minimum are given by, respectively, $I_{max} = \frac{C_0^2\Gamma^2(n+1)}{n+1}\left[\frac{nP_{cg}}{(n+1)(P_{cg}-P_0)}\right]^n$ and $I_{min} = (C_0 n!)^2 \frac{P_0}{P_{cg}}$. If we define the depth of the concave as $I_{depth} = I_{max} - I_{min}$, one can calculate the depth easily and find that a weaker beam power leads to a greater depth (see Fig. 1(a) and (b)).

(b) The platform case. When $P_0 = P_{cp}^{(n)}$, for the evolution curve of on-axis intensity, there exists a platform in each period. This indicates the on-axis intensity can almost keep invariant during a certain propagation distance (see Fig. 1(c)).

(c) The convex case. When $P_0 > P_{cp}^{(n)}$, the evolution curve of on-axis intensity changes into a hump in each period (see Fig. 1(d)). As a result, there only exists one extreme value of on-axis intensity (i.e. one maximum) in each period, and it is located in the middle of each period, i.e. $z = (2q+1)\pi\sqrt{P_{cg}/P_0}z_R/2$. The maximal value of on-axis intensity is $I_{max} = (C_0 n!)^2 P_0/P_{cg}$, which corresponds to the minimum for the concave case.

Fig. 2 shows the effect of the beam order on the on-axis intensity of HBs in HNNM. One can find that, for a fixed beam power, if an intensity concave comes into being, the depth of the concave decreases with the beam order increasing (see Fig. 2(a)). Namely, the maximal intensity becomes smaller and smaller, and the minimal intensity becomes larger and larger with the beam order increasing until the concave disappears. And then the intensity convex occurs. The maximal intensity appears at the middle of each period (see Fig. 1(d)) and increases with increasing beam order (see Fig. 2(b)).

One can find from Fig. 1 that the longer the period, the lower the beam power. This is because a weaker beam power provides a weaker self-focusing. Based on the expression of on-axis intensity, one can get the evolution period of HBs in HNNM is [8] $\Delta z = \pi\sqrt{P_{cg}/P_0}z_R$. In addition, compared with the modified hollow Gaussian beams in HNNM [19], it is found that the evolutions of on-axis intensity are similar, therefore we think these results in this paper are suitable for other non-vortical hollow beams.

In conclusion, we have investigated the on-axis intensity evolutions of HBs in HNNM, and a set of analytical results are given. Three types of on-axis intensity evolution induced by different beam powers are illustrated in detail.

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